

پاسخنامه تشریحی

$$\log_a x = \frac{f}{g} \rightarrow x = a^{\frac{f}{g}} \rightarrow x = (a^g)^{\frac{f}{g}} \rightarrow x = a^f \rightarrow \boxed{x = 16} \rightarrow \sqrt{x} = \sqrt{16} \rightarrow \boxed{\sqrt{x} = 4}$$

$$\log_{16} x = \frac{1}{8} \rightarrow x = 16^{\frac{1}{8}} \rightarrow x = (2^4)^{\frac{1}{8}} \rightarrow x = 2^{\frac{1}{2}} \rightarrow \boxed{x = \sqrt{2}}$$

$$\log_9 18 = \log_9 (3^2 \times 2) = \log_9 3^2 + \log_9 2 = 2 \log_9 3 + \log_9 2 = 2 + \log_9 2$$

$$\rightarrow 2 + \log_9 2 = a \rightarrow \boxed{\log_9 2 = a - 2}$$

$$\log_a x = b \rightarrow \boxed{x = a^b}, \quad \log_y x = \frac{b}{c} \rightarrow x = y^{\frac{b}{c}}$$

$$\rightarrow x^{\frac{c}{b}} = (y^{\frac{b}{c}})^{\frac{c}{b}} \rightarrow y = x^{\frac{c}{b}} \rightarrow y = (a^b)^{\frac{c}{b}} \rightarrow \boxed{y = a^c}$$

$$\log \frac{125}{100} = \log \frac{125}{100} = \log 125 - \log 100 = \log 5^3 - \log 10^2$$

$$= 3 \log 5 - 2 \log 10 = 3(1 - a) - 2(1) = 1 - 3a$$

$$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 \rightarrow \log 5 = 1 - a$$

$$\log_{\sqrt{2}} 125 = \frac{\log 125}{\log \sqrt{2}} = \frac{\log 5^3}{\log (2^{\frac{1}{2}})} = \frac{3 \log 5}{\log 2^{\frac{1}{2}} + \log 2^{\frac{1}{2}}} = \frac{3 \log 5}{\frac{1}{2} \log 2 + \frac{1}{2} \log 2} = \frac{3(1 - a)}{1} = \frac{3 - 3a}{1}$$

$$f(42) = 3 - 2 \log_9 \left(\frac{42}{9} - 5 \right) = 3 - 2 \log_9 (21 - 5) = 3 - 2 \log_9 16$$

$$= 3 - 2 \log_9 4^2 = 3 - 2 \times 2 \log_9 4 = 3 - 4 = -1$$

$$\begin{cases} \log_9 x + \log_9 y = 2 \\ x^2 + y^2 = 46 \end{cases} \rightarrow \begin{cases} \log_9 xy = 2 \\ x^2 + y^2 = 46 \end{cases} \rightarrow \begin{cases} xy = 3^2 \\ x^2 + y^2 = 46 \end{cases} \rightarrow \begin{cases} xy = 9 \\ x^2 + y^2 = 46 \end{cases}$$

$$\rightarrow (x + y)^2 = x^2 + y^2 + 2xy \rightarrow (x + y)^2 = 46 + 2(9) \rightarrow (x + y)^2 = 64$$

$$\begin{cases} \boxed{x + y = 8} \\ x + y = -8 \end{cases} \rightarrow \log_9 (x + y) = \log_9 8 = \log_9 2^3 = \frac{3}{2} \log_9 2 = \frac{3}{2} \rightarrow \boxed{\log_9 (x + y) = \frac{3}{2}}$$

$$\log_x (x^2 + 4) = \log_x x + \log_x 4 \rightarrow \log_x (x^2 + 4) = \log_x 4x \rightarrow x^2 + 4 = 4x$$

$$\rightarrow x^2 - 4x + 4 = 0 \rightarrow (x - 1)(x - 4) = 0 \rightarrow \begin{cases} x - 1 = 0 \rightarrow x = 1 \text{ غ} \\ x - 4 = 0 \rightarrow \boxed{x = 4} \end{cases}$$

$$\rightarrow \log_9 x = \log_9 4 = \log_9 2^2 = 2 \log_9 2 = 2 \rightarrow \boxed{\log_9 x = 2}$$

$$\text{الف) } \log_9 (x^2 - 24) = \log_9 5x \rightarrow x^2 - 24 = 5x \rightarrow x^2 - 5x - 24 = 0$$

$$\rightarrow (x - 8)(x + 3) = 0 \rightarrow \begin{cases} x - 8 = 0 \rightarrow \boxed{x = 8} \\ x + 3 = 0 \rightarrow x = -3 \text{ غ} \end{cases}$$

$$\text{ب) } \log_9 (x^2 - 1) = 1 + \log_9 (x + 9) \rightarrow \log_9 (x^2 - 1) = \log_9 3 + \log_9 (x + 9)$$

$$\rightarrow \log_9 (x^2 - 1) = \log_9 (3x + 27) \rightarrow x^2 - 1 = 3x + 27 \rightarrow x^2 - 3x - 28 = 0$$

$$\rightarrow (x - 7)(x + 4) = 0 \begin{cases} x - 7 = 0 \rightarrow x = 7 \\ x + 4 = 0 \rightarrow x = -4 \end{cases}$$

پ) $\log_x(x^7 - 2x) = 2 \rightarrow x^7 - 2x = x^2 \rightarrow x^7 - x^2 - 2x = 0$

$$\rightarrow x(x^6 - x - 2) = 0 \rightarrow x(x - 2)(x + 1) = 0 \begin{cases} x = 0 \text{ غ} \\ x - 2 = 0 \rightarrow x = 2 \\ x + 1 = 0 \rightarrow x = -1 \text{ غ} \end{cases}$$

ت) $\begin{cases} \log x = \log 2 + \log y \\ 2^x \times 8^y = 4 \end{cases} \rightarrow \begin{cases} \log x = \log 2y \\ 2^x \times (2^3)^y = 2^2 \end{cases} \rightarrow \begin{cases} x = 2y \\ x + 3y = 2 \end{cases}$

جایگذاری $\rightarrow 2y + 3y = 2 \rightarrow 5y = 2 \rightarrow y = \frac{2}{5}, x = \frac{4}{5}$

ث) $\begin{cases} \log y = 2 \log 3 + \log x \\ 2^{x-7} \times 4^{x+y} = 1 \end{cases} \rightarrow \begin{cases} \log y = \log 3^2 + \log x \\ 2^{x-7} \times (2^2)^{x+y} = 2^0 \end{cases} \rightarrow \begin{cases} \log y = \log(9x) \\ 2^{x-7} \times 2^{2x+2y} = 2^0 \end{cases}$

جایگذاری $\rightarrow \begin{cases} y = 9x \\ 3x + 2y = 7 \end{cases} \rightarrow 3x + 2(9x) = 7 \rightarrow x = \frac{1}{3}, y = 3$

$$\log_p(2x^7 + 1) - \log_p(x + 2) = 1 \rightarrow \log_p \frac{2x^7 + 1}{x + 2} = \log_p 3 \rightarrow \frac{2x^7 + 1}{x + 2} = 3$$

$$\rightarrow 2x^7 + 1 = 3x + 6 \rightarrow 2x^7 - 3x - 5 = 0 \rightarrow (x + 1)(2x - 5) = 0 \begin{cases} x + 1 = 0 \rightarrow x = -1 \\ 2x - 5 = 0 \rightarrow x = \frac{5}{2} \end{cases}$$

$x = -1 \rightarrow \log_8(2x - 1) = \log_8(2(-1) - 1) = \log_8(-3)$ غ

$$x = \frac{5}{2} \rightarrow \log_8(2x - 1) = \log_8(2(\frac{5}{2}) - 1) = \log_8 4 = \log_8 2^2 = \frac{2}{3} \log_8 2 \rightarrow \log_8(2x - 1) = \frac{2}{3}$$

$$I_0 = 10^{-12} \text{ wat} \frac{t}{m^2}, I = 2 \times 10^{-10} \text{ wat} \frac{t}{m^2}$$

$$\rightarrow D = 10 \log \frac{I}{I_0} = 10 \log \left(\frac{2 \times 10^{-10}}{10^{-12}} \right) = 10 \log(2 \times 100) = 10(\log 2 + \log 100)$$

$$\rightarrow D = 10(0,3010 + 2) = 10(2,301) \rightarrow D = 23,01 \text{ db}$$

جمعیت پس از t سال ، زمان t ، رشد سالانه k ، جمعیت اولیه A ،

$$\rightarrow f(t) = A(1 + k)^t \rightarrow f(t) = 100000(1 + 0,04)^t \rightarrow f(t) = 100000(1,04)^t$$

الف) $t = 0 \rightarrow p(0) = 600 \times 2^0 \rightarrow p(0) = 600$

ب) $t = 8 \rightarrow p(8) = 600 \times 2^{\frac{8}{2}} = 600 \times 2^4 = 600 \times 16 \rightarrow p(8) = 9600$

پ) $p(t) = 76800 \rightarrow 600 \times 2^{\frac{t}{2}} = 76800 \rightarrow 2^{\frac{t}{2}} = \frac{76800}{600} \rightarrow 2^{\frac{t}{2}} = 128$

$$\rightarrow 2^{\frac{t}{2}} = 2^7 \rightarrow \frac{t}{2} = 7 \rightarrow t = 14 \text{ ساعت}$$

$$\log E = 11,8 + 1,5M \rightarrow \log(10^{22,6}) = 11,8 + 1,5M$$

$$\rightarrow 22,6 = 11,8 + 1,5M \rightarrow 22,6 - 11,8 = 1,5M \rightarrow 10,8 = 1,5M \rightarrow M = 7,2 \text{ ریشتنر}$$